Proceedings of the 39th International Conference on Ocean, Offshore and Arctic Engineering
OMAE2020
June 28 - July 3, 2020, Fort Lauderdale, Florida, USA

OMAE2020-8219

GLOBAL HIERARCHICAL MODELS FOR WIND AND WAVE CONTOURS: PHYSICAL INTERPRETATIONS OF THE DEPENDENCE FUNCTIONS

Andreas F. Haselsteiner*
University of Bremen
Bremen, Germany
Email: a.haselsteiner@uni-bremen.de

Aljoscha Sander
University of Bremen
Bremen, Germany
Email: alsander@uni-bremen.de

Jan-Hendrik Ohlendorf
University of Bremen
Bremen, Germany
Email: johlendorf@uni-bremen.de

Klaus-Dieter Thoben
University of Bremen
Bremen, Germany
Email: thoben@uni-bremen.de

ABSTRACT

Applications such as the design of offshore wind turbines requires the estimation of the joint distribution of variables like wind speed, wave height and wave period. The joint distribution can then be used, for example, to define design load cases using the environmental contour method. Often the joint distribution is described using so-called global hierarchical models. In these models, one variable is taken as independent and the other variables are modelled to be conditional on this variable using particular dependence functions. In this paper, we propose to use dependence functions that offer physical interpretation. We define a novel dependence function that describes how the median of the zero-up-crossing period increases with significant wave height and a novel dependence function that describes how the median significant wave height increases with wind speed. These dependence functions allow us to reason about the physical meaning, even when we extrapolate outside the range of a given sample of environmental data. In addition, we can analyze the estimated parameters of the dependence function to speculate which kind of sea dominates at a given site. We fitted statistical models with the proposed dependence functions to six datasets and analyzed the estimated parameters. Then we calculated environmental contours based on these estimated joint distributions. The environmental contours had physically reasonable shapes, even at areas that were outside the datasets that were used to fit the underlying distributions.

INTRODUCTION

Environmental contours are used to define extreme environmental conditions for which a structure such as an offshore wind turbine can be evaluated. They describe joint extremes of environmental variables like wave height, wave period, wind speed or current. The process of constructing an environmental contour – the environmental contour method – is widely used in the process of designing offshore structures and standards and guidelines like IEC 61400-3-1:2019 [1] and DNV-RP-C205:2017 [2] recommend the use of the method.

Similar to other important numerical design methods there exist a variety of different specific environmental contour methods. Differences can arise due to (i) different environmental modelling techniques, (ii) different mathematical definitions of which area in the variable space a contour of a given probability should cover and (iii) which algorithms are used to compute a contour. Despite the method's popularity and much research on it within the last years (for a review, see [3]), common state-of-the-art environmental contour methods can produce non-

*Address all correspondence to this author.

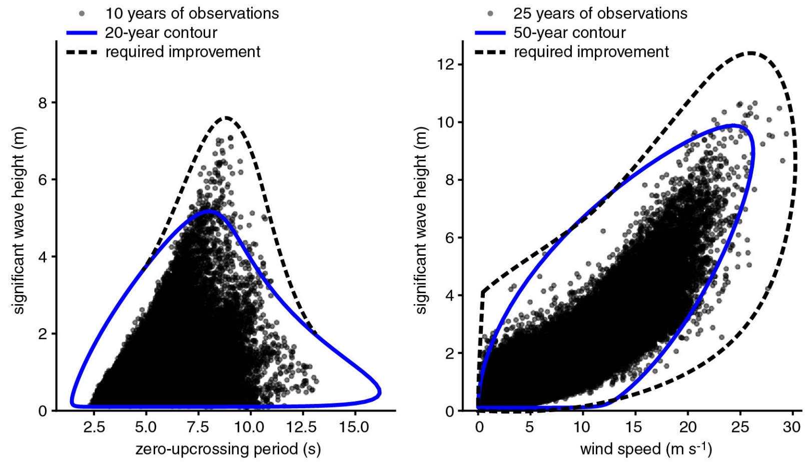

Figure 1. PROBLEMS APPARENT IN STATE-OF-THE-ART ENVIRONMENTAL CONTOURS AND THEIR STATISTICAL MODELS. (A) SEA STATE CONTOURS EXCLUDE HIGH SEA STATES. (B) WIND-WAVE CONTOURS EXCLUDE HIGH WIND AND HIGH WAVE STATES AND FOLLOW AN INCORRECT SHAPE. THE TWO SHOWN CONTOURS (CONTINUOUS LINES) WERE PRESENTED IN A RECENT BENCHMARKING STUDY [4].

conservative contours. A recent benchmarking study [4] that was co-organized by the first author of this paper, showed such contours. In the study, the authors used joint models for sea states and for wind-wave states that are recommended in current engineering guidelines and intentionally applied them without checking whether the models were appropriate for the particular offshore site. As the fitted joint distributions did not catch the data structure properly, environmental contours based on these distributions had incorrect shapes (Fig. 1): The sea state contour predicted sea states with too low significant wave height and the wind-wave contour described an incorrect shape.

Most joint distributions that are used to describe the metocean environment are not based on physical models. For example, the marginal distribution of the significant wave height is sometimes modelled with a 2-parameter Weibull distribution [5], a 3-parameter Weibull distribution [6], a gamma distribution [7] or a hybrid lognormal-Weibull distribution ('Lonowe model') [8] and there is no physical model that supports the use of a particular distribution. Similarly, the dependence structure between the metocean variables does not offer physical interpretation. In global hierarchical models, joint distributions are built up using conditional parametric distributions. For example, significant wave height, H_s, might be modelled with a marginal 3-parameter Weibull distribution and wind speed, V, with a 2-parameter Weibull distribution whose parameters depend on the value of H_s. Then so-called dependence functions are used, which in the given case might be $\alpha_V(hs) = c_1 + c_2 h_s^{c_3}$ where α_V represents the wind distribution's scale parameter. Such a dependence function might fit well to a particular dataset, however, it does not provide direct physical insights. As one cannot physically interpret such a dependence function, reasoning how well a particular fitted dependence function extrapolates outside a dataset could be called statistically informed guessing.

This work was motivated by the potential advantages that dependence functions that can be interpreted physically might have. Much is known about how winds and waves behave: how wind generates waves, when waves break and how wind sea and swell mix. If we can utilize this knowledge in the formalization of the joint model of environmental variables we should be able to design models that extrapolate better and whose parameters can be interpreted physically. In this paper, we will design a novel dependence structure for the joint distribution of significant wave height and zero-up-crossing period, $H_s - T_z$, and a novel dependence structure for wind speed and wave height, $V - H_s$. We will focus on using physically interpretable expressions that describe how T_z depends on H_s and how H_s depends on V. Then, we will estimate the parameters of these distributions by fitting the models to six datasets describing metocean conditions in the Atlantic and in the North Sea. Finally, we will compute environmental contours based on the fitted joint distributions.

DATA AND METHODS

Datasets

To test our models, we used six datasets: three datasets that describe sea states (H_s-T_z, datasets A, B, C) and three datasets that describe wind-wave states (V-H_s, datasets D, E, F; Tab. 1). These datasets were provided in a recent benchmarking study on estimating extreme environmental conditions [4] and are available at a GitHub repository[1]. Datasets A, B and C are from buoys of the National Data Buoy Center [9] and cover 10 years of hourly sea states. Datasets D, E and F were retrieved from

[1] https://github.com/ec-benchmark-organizers/ec-benchmark

the hindcast coastDat-2 [10] and cover 25 years of hourly wind and wave data. The wind data represent a 10-minute mean value, measured 10 m above sea level.

Table 1. USED DATASETS.

Dataset	Variables	Site	Data source
A	H_s, T_z	off Maine coast	buoy 44007 [9]
B	H_s, T_z	off Florida coast	buoy 41009 [9]
C	H_s, T_z	Gulf of Mexico	buoy 42001 [9]
D	V, H_s	off German coast	hindcast [10]
E	V, H_s	off UK coast	hindcast [10]
F	V, H_s	off Norwegian coast	hindcast [10]

Global hierarchical models

Global hierarchical models are probabilistic models that cover the complete range of an environmental variable ('global') and which follow a particular hierarchical dependence structure. In a global hierarchical model, if the joint density function is factorized, simple terms for the univariate density functions exist. Let X_1 and X_2 represent random variables, for example $X_1 = V$ and $X_2 = H_s$, and let $f_{X1,X2}(x_1,x_2)$ represent its joint density function. Then the factorization

$$f_{X1,X2}(x_1,x_2) = f_{X1}(x_1) f_{X2|X1}(x_2|x_1) \tag{1}$$

describes a hierarchy where a random variable with index i can only depend upon random variables with indices less than i. Usually, simple parametric distributions are assumed for the random variables and the dependence of X_2 under X_1 is modelled using simple dependence functions with 2-4 parameters (see, for example, [11, 12]). Let α_2 and β_2 represent the parameters of the second distribution. Then these parameters might be modelled with dependence functions with n parameters:

$$\begin{aligned} \alpha_2 &= f_\alpha(x_1; c_1, c_2, ..., c_n), \\ \beta_2 &= f_\beta(x_1; c_1, c_2, ..., c_n). \end{aligned} \tag{2}$$

Typical expressions are a power function, $f_\alpha(x_1) = c_1 + c_2 x_1^{c_3}$, or an exponential function, $f_\alpha(x_1) = c_1 + c_2 e^{c_3 x_1}$ [2].

We assumed the following model for sea states: Significant wave height follows an exponentiated Weibull distribution and zero-up-crossing period follows a log-normal distribution that depends on the value of H_s. The exponentiated Weibull distribution has been recently proposed to model H_s [13] and the log-normal distribution is a usual choice for modelling $T_z|H_s$. The two distributions read

$$\begin{aligned} F(h_s) &= \left(1 - \exp\left[-(h_s/\alpha)^\beta\right]\right)^\delta, \\ F(t_z|h_s) &= \frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\ln t_z - \mu_{tz}}{\sqrt{2}\sigma^2}\right). \end{aligned} \tag{3}$$

We modelled the log-normal distribution's parameter μ_{tz} with a two-parameter dependence function:

$$\mu_{tz} = \ln\left(c_1 + c_2\sqrt{\frac{h_s}{g}}\right), \tag{4}$$

where g is the gravity constant, $g = 9.81\,\mathrm{m\,s^{-2}}$ and c_1 and c_2 are parameters that will be estimated. The parameter μ_{tz} can be directly related to the period's median, $\tilde{t}_z = e^{\mu_{tz}}$. Thus, the dependence function implies

$$\tilde{t}_z = c_1 + c_2\sqrt{\frac{h_s}{g}}, \tag{5}$$

which is an expression that is physically consistent as $[\tilde{t}_z] = \mathrm{s}$, $[h_s] = \mathrm{m}$ and $[g] = \mathrm{m\,s^{-2}}$ if $[c_1] = \mathrm{m}$ and c_2 is unitless. The parameter σ_{tz} is modelled with an asymptotically decreasing dependence function:

$$\sigma_{tz} = c_3 + \frac{c_4}{1 + c_5 h_s}. \tag{6}$$

For the wind-wave joint distribution the following model is assumed: Wind speed follows an exponentiated Weibull distribution and significant wave height follows an exponentiated Weibull distribution that is conditional on the value of V:

$$\begin{aligned} F(v) &= \left(1 - \exp\left[-(v/\alpha)^\beta\right]\right)^\delta, \\ F(h_s|v) &= \left(1 - \exp\left[-(h_s/\alpha_{hs})^{\beta_{hs}}\right]\right)^{\delta_{hs}} \end{aligned} \tag{7}$$

The Weibull distribution is a typical choice for both, wind speed and wave height. However, usually wave height is modelled with a translated Weibull distribution and wind speed with a 2-parameter Weibull distribution that depends on the value of H_s (see, for example, [2, 4]) instead of the exponentiated Weibull distribution.

We model the dependence structure between the two variables by assuming that the median of H_s increases with wind speed and that the shape parameter β_{hs} follows a logistics function:

$$\tilde{h}_s = c_6 + c_7 v^{c_8}, \\ \beta_{hs} = c_9 + \frac{c_{10}}{1 + e^{-c_{11}(v - c_{12})}}. \tag{8}$$

The exponent of the exponentiated Weibull distribution is set to $\delta_{hs} = 5$ such that the dependence function for the scale parameter α_{hs} reads

$$\alpha_{hs} = (c_6 + c_7 v^{c_8})/2.0445^{1/\beta_{hs}}. \tag{9}$$

This structure has the advantage that the relationship between typical (median) wave height values and wind speed values is modelled with a simple expression that can be interpreted physically. The estimated exponent c_8 might imply that $\tilde{h}_s$ increases linearly, quadratically or something in between with increasing wind speed and its value can be compared with theories on wind-generated seas (see, for example, [14])

Parameter estimation and contour computation

The eight parameters of the sea state model and the ten parameters of the wind-wave model were estimated by fitting the described model structure to each of the datasets. All computations were performed using the open-source Python software viroconcom in version 1.3.9 [15].

We fitted the sea state model by following a step-wise process: First the marginal distribution of H_s was fitted by using a weighted least squares method that prioritizes high quantiles of H_s (for details, see [13]). Second, zero-up-crossing period was sorted into H_s intervals. We used an interval size of 0.5 m. Third, marginal distributions were fitted to zero-up-crossing period in each interval that held at least 50 data points using maximum likelihood estimation. Finally, the dependence functions were fitted using nonlinear least squares.

Similarly, the parameters of the wind-wave models were estimated following a step-wise process: The marginal distributions of V and H_s were fitted using a weighted least squares method [13]. The interval size was $2\,\mathrm{m\,s^{-1}}$ and the required number of datapoints within each bin was set to 50 as well. The dependence functions were fitted using nonlinear least squares with weights of 1 / parameter value such that errors were normalized.

The resulting six joint distributions were used to compute highest density environmental contours. A highest density contour is one possible definition for an environmental contour [16] that in principle can also be used for setting design requirements of non-environmental variables [17]. It is based on the statistical concept of the highest density region [18]. A highest density contour, C, encloses a highest density region, R, that holds probability $1-\alpha$. As the border of a highest density region has constant probability density, the contour can be expressed as the set of all environmental states $\mathbf{x}$ whose probability density equals the density value f_c:

$$C(\alpha) = \{\mathbf{x} \in \mathbb{R}^d : f(\mathbf{x}) = f_c\}. \tag{10}$$

The challenge of constructing such a contour lies in computing the density value f_c that corresponds to a particular exceedance probability α. In any environmental contour method, the exceedance probability α relates to the return period of interest as

$$\alpha = \frac{T_S}{T_R}, \tag{11}$$

where T_S is the environmental state duration and T_R is the return period of interest. In this study, we considered two-dimensional environmental states: either sea states, $\mathbf{x} = (h_s, t_z)$, or wind-wave states, $\mathbf{x} = (v, h_s)$.

As prescribed in a recent benchmarking exercise [4], we computed 1-year and 20-year environmental contours with our sea state models and 1-year and 50-year contours with our wind-wave models. The scripts that were used to fit the joint distributions and to compute the contours are available at a GitHub repository[2].

RESULTS

Statistical models: Dependence functions

Table 2 presents the estimated parameters of the statistical models. The fitted sea state models have common characteristics, which can be derived from the estimated parameter values (Figure 2). The dependence functions for the parameter μ_{tz} imply a relationship between T_z and H_s of

$$\tilde{t}_z(h_s) = [2.7,\, 3.6]\,\mathrm{s} + [5.3,\, 6.5]\sqrt{h_s/g}, \tag{12}$$

where values within the brackets hold the lowest and highest parameter values among the three datasets. For high sea states, $h_s = 10$ m, these dependence functions predict $\tilde{t}_z = \{9.5\,\mathrm{s}, 8.9\,\mathrm{s}, 9.3\,\mathrm{s}\}$ for datasets A, B and C, respectively. Visually comparing these predicted sea states with the datasets suggest that the dependence functions are reasonable (Figure 8).

[2]https://github.com/ahaselsteiner/2020-paper-omae-hierarchical-models

Table 2. FITTED STATISTICAL MODELS.

Dataset	**Significant wave height**			**Zero-upcrossing period, log-normal distribution**						
	Exponentiated Weibull distribution			$\mu_{tz}(h_s) = \ln\left(c_1 + c_2\sqrt{\frac{h_s}{g}}\right)$			$\sigma_{tz}(h_s) = c_3 + \frac{c_4}{1 + c_5 h_s}$			
	α (scale)	β (shape)	δ (shape)	c_1	c_2		c_3	c_4	c_5	
A	0.207	0.684	7.79	3.62	5.77		0	0.324	0.404	
B	0.0988	0.584	36.6	3.54	5.31		0	0.241	0.256	
C	0.227	0.697	9.85	2.71	6.51		0.109	0.147	0.236	
	Wind speed			**Sig. wave height, exp. Weibull distribution with $\delta = 5$**						
	Exponentiated Weibull distribution			$\alpha_{hs}(v) = (c_6 + c_7 v^{c_8})/2.0445^{1/\beta_{hs}(v)}$			$\beta_{hs}(v) = c_9 + \frac{c_{10}}{1 + e^{-c_{11}(v - c_{12})}}$			
	α (scale)	β (shape)	δ (scale)	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}
D	10.0	2.42	0.761	0.488	0.0114	2.03	0.714	1.70	0.304	8.77
E	10.8	2.48	0.683	0.617	0.0174	1.87	0.724	2.01	0.309	9.59
F	11.5	2.56	0.534	1.09	0.0251	1.80	0.726	1.89	0.194	13.4

These dependence functions can also be used to analyze what they imply for steepness. Steepness, s, is a non-dimensional variable that describes a sea state [14, p. 88]:

$$s = \frac{2\pi h_s}{g t_z^2}. \tag{13}$$

Using our expression for $\tilde{t}_z$, we can derive a model for the expected median steepness, $\tilde{s}$:

$$\tilde{s} = \frac{2\pi h_s}{g(c_1 + c_2\sqrt{h_s/g})^2}. \tag{14}$$

A plot of steepness over significant wave height suggests that the predicted median steepness is reasonable (Fig. 3). Interestingly, the used datasets contain many datapoints whose steepness exceeds $1/15$, which is sometimes seen as an upper limit due to wave breaking (see, for example, [14, p. 88]).

Similarly, the dependence functions of the fitted wind-wave models show similarities among the three datasets and can be physically interpreted (Figure 5). The median of significant wave height increases with wind speed in the following manner:

$$\tilde{h}_s = [0.5, 1.1]\,\mathrm{m} + [0.011, 0.025] v^{[1.8, 2.0]}, \tag{15}$$

where values within the brackets hold the lowest and highest coefficient values among the three datasets. This suggests that there are two parts to significant wave height: one part that is independent of local wind speed and that is in the order of 1 m and one part that increases with wind speed, where the increase is more than linear, but at two sites also less than quadratic. The first part could be interpreted as either a swell component or as waves that were generated locally, but in the past. The second part is especially interesting as it might offer insights into the nature of the sea state. The found exponents between 1 and 2 lie between the limits of two different kind of seas: In fully-developed wind-generated seas that follow a Pierson-Moskowitz spectrum, significant wave height quadratically increases with wind speed [19, p. 35]. In seas, which are not fully developed, significant wave height might increase linearly (if they are fetch-limited) or with $v^{9/7}$ if they are duration-limited [20, p. 360].

Exponentiated Weibull distribution

The exponentiated Weibull distribution is a novel distribution choice for sea state models and for wind-wave models. The marginal distribution of H_s was modelled well with the exponentiated Weibull distribution and a detailed analysis on its goodness of fit for datasets A, B and C was presented in a recent publication by this paper's first author [13].

In the wind-wave models, we used the exponentiated Weibull distribution to model the marginal distribution of wind speed. Usually, a 2-parameter Weibull distribution is used instead, which is an exponentiated Weibull distribution with $\delta = 1$. Thus, thanks to its additional parameter, an exponentiated Weibull distribution will fit any dataset at least as good as a 2-parameter Weibull distribution. However, the additional parameter adds complexity and thus its use over the 2-parameter Weibull distribution should be justified. The distribution of wind speed in datasets D and E can be described well with a 2-parameter

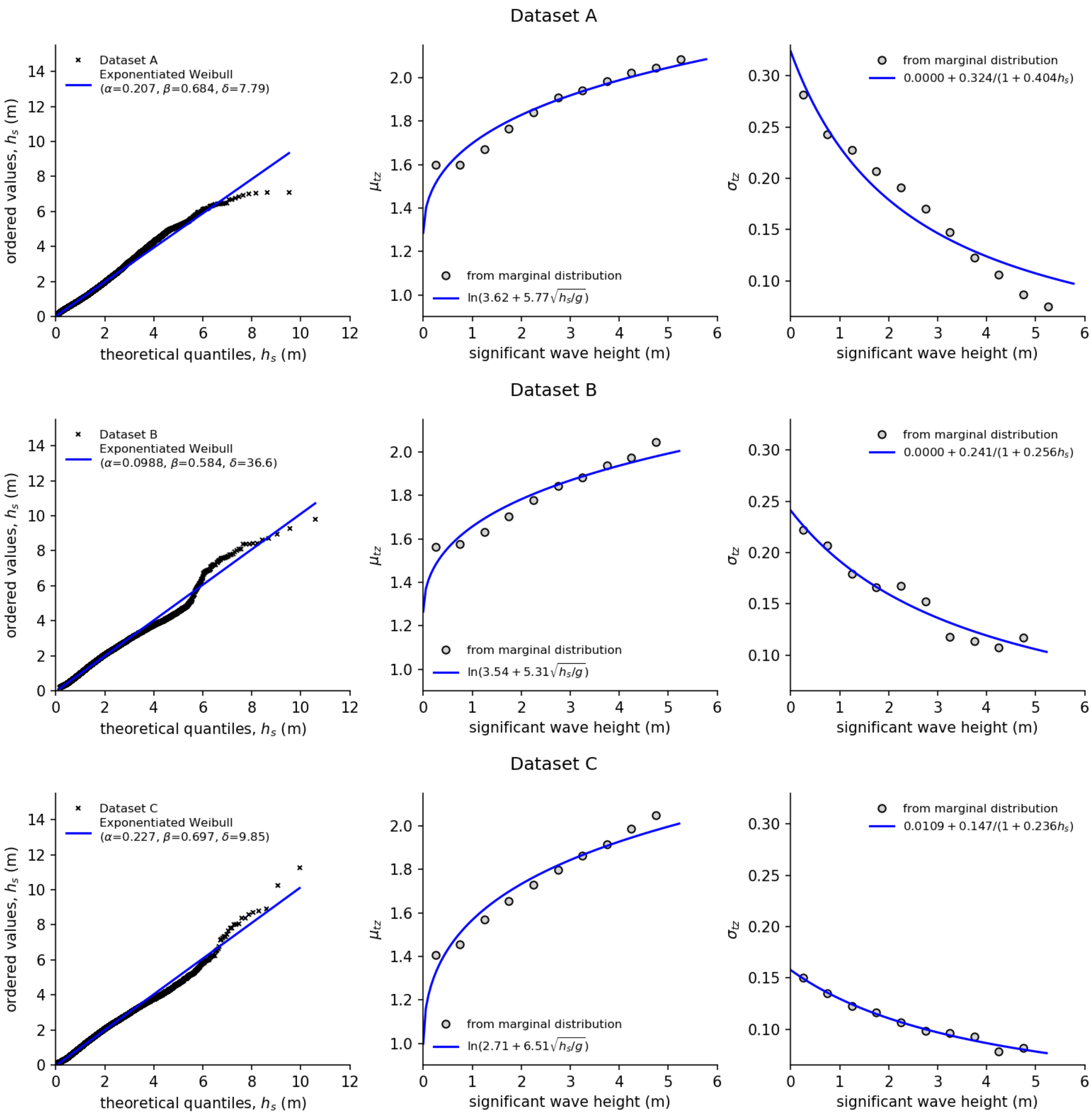

Figure 2. FITS OF THE SEA STATE MODEL. THE DEPENDENCE FUNCTION OF μ_{tz} IS DESIGNED SUCH THAT THE MEDIAN OF THE ZERO-UP-CROSSING PERIOD INCREASES WITH $\sqrt{h_s/g}$ WHERE h_s REPRESENTS SIGNIFICANT WAVE HEIGHT AND g REPRESENTS EARTH'S GRAVITY CONSTANT.

Weibull distribution, however, in dataset F the data do not follow a 2-parameter Weibull distribution at high wind speeds (Fig. 4). A fitted 2-parameter Weibull distribution would predict too high wind speeds in dataset F while a fitted exponentiated Weibull distribution can follow the shape of the empirical distribution and thus can predict wind speeds better (Figure 5 left)

Finally, we used the exponentiated Weibull distribution to model the distribution of significant wave height within given wind speed intervals. To keep model complexity – measured in numbers of free parameters – in balance, we used an exponentiated Weibull distribution with a fixed exponent of $\delta = 5$. This choice was based on first fitting exponentiated Weibull distributions with free exponents to binned wave data. We found that δ varied between 2 and 23 and that it followed a bell-like curve (Figure 6). Based on these results we decided to set $\delta = 5$ and to fit these fixed-exponent distributions again such that we would

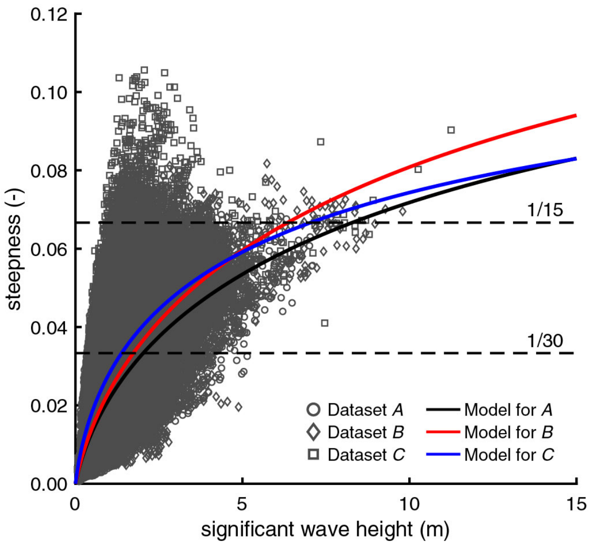

Figure 3. THE DEPENDENCE FUNCTION THAT DESCRIBES HOW ZERO-UP-CROSSING PERIOD CHANGES WITH WAVE HEIGHT CAN BE USED TO DERIVE THE PREDICTED RELATIONSHIP FOR THE MEDIAN OF STEEPNESS AS A FUNCTION OF WAVE HEIGHT.

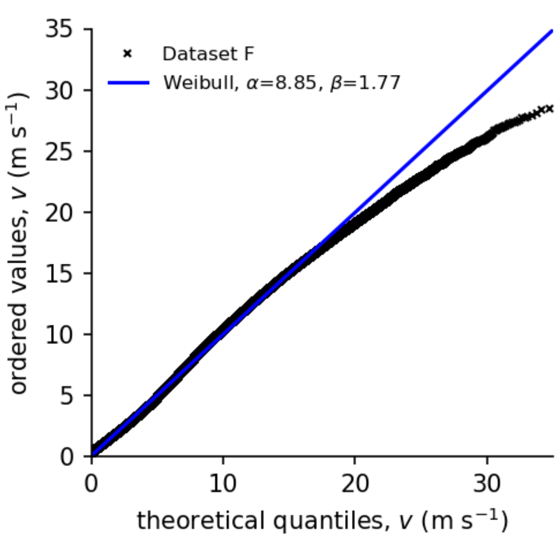

Figure 4. IN DATASET F WIND SPEED DOES NOT FOLLOW A TWO-PARAMETER WEIBULL DISTRIBUTION AT HIGH QUANTILES.

get the parameter values of α and β that lead to the best fit with $\delta = 5$.

The exponentiated Weibull distributions with $\delta = 5$ showed good model fit at all wind speed intervals (Fig. 7). Plotting the distribution on Weibull paper illustrates that a 2-parameter Weibull distribution is insufficient to describe the empirical distribution. The second shape parameter of the exponentiated Weibull distribution, $\delta = 5$, however, enables the distribution to follow the shape of the data. These results suggest that if data do not indicate otherwise, conditional significant wave height data should be assumed to follow an exponentiated Weibull distribution.

Environmental contours

Both, the sea state and the wind-wave contours seem to catch the two-dimensional structure of the datasets (Figure 8). Similar to the observations, the contours of dataset A and B have a triangular-like shape. Dataset C does not hold sea states with long periods, but low intensity and consequently the contours of dataset C do not have a triangular-like shape. In dataset A the 20-year contour includes all observations of the 10-year long dataset. In dataset B the 20-year contour excludes 30 datapoints and in dataset C the contour excludes 6 datapoints.

While the shapes of the contours of dataset D and F look reasonable overall, the 50-year contour of dataset E has a questionable region at low wind speeds: The 50-year contour predicts slightly higher sea states at $v = 1\,\mathrm{m\,s^{-1}}$ than at $v = 5\,\mathrm{m\,s^{-1}}$. While the 50-year contour of dataset F includes all observations of the 25-year long dataset, the contours of datasets D and E both exclude 2 datapoints.

CONCLUSIONS

In this paper, we showed how joint models of $H_s - T_z$ and $V - H_s$ can be designed such that the dependence function of the conditional variable offers physical interpretation. We modelled the median of the zero-up-crossing period to increase with $\sqrt{h_s}$ and the median of the significant wave height to increase with v^c where the exponent c was estimated to be between 1.8 and 2. The relationship $\tilde{t}_z \propto \sqrt{h_s}$ ensures that the wave period increases in an interpretable physical manner and the relationship $\tilde{h}_s \propto v^c$ offers insights into which kind of sea the joint distribution describes. Finally, we computed environmental contours based on these joint distributions. Overall, the environmental contours had reasonable shapes and contours with long return periods extrapolated in a physically interpretable manner: They behaved similar to the relationships of the median period conditional on wave height, $\tilde{T}_z|H_s$, and of the median wave height conditonal on wind speed, $\tilde{H}_s|V$, that were expressed in the fitted dependence functions.

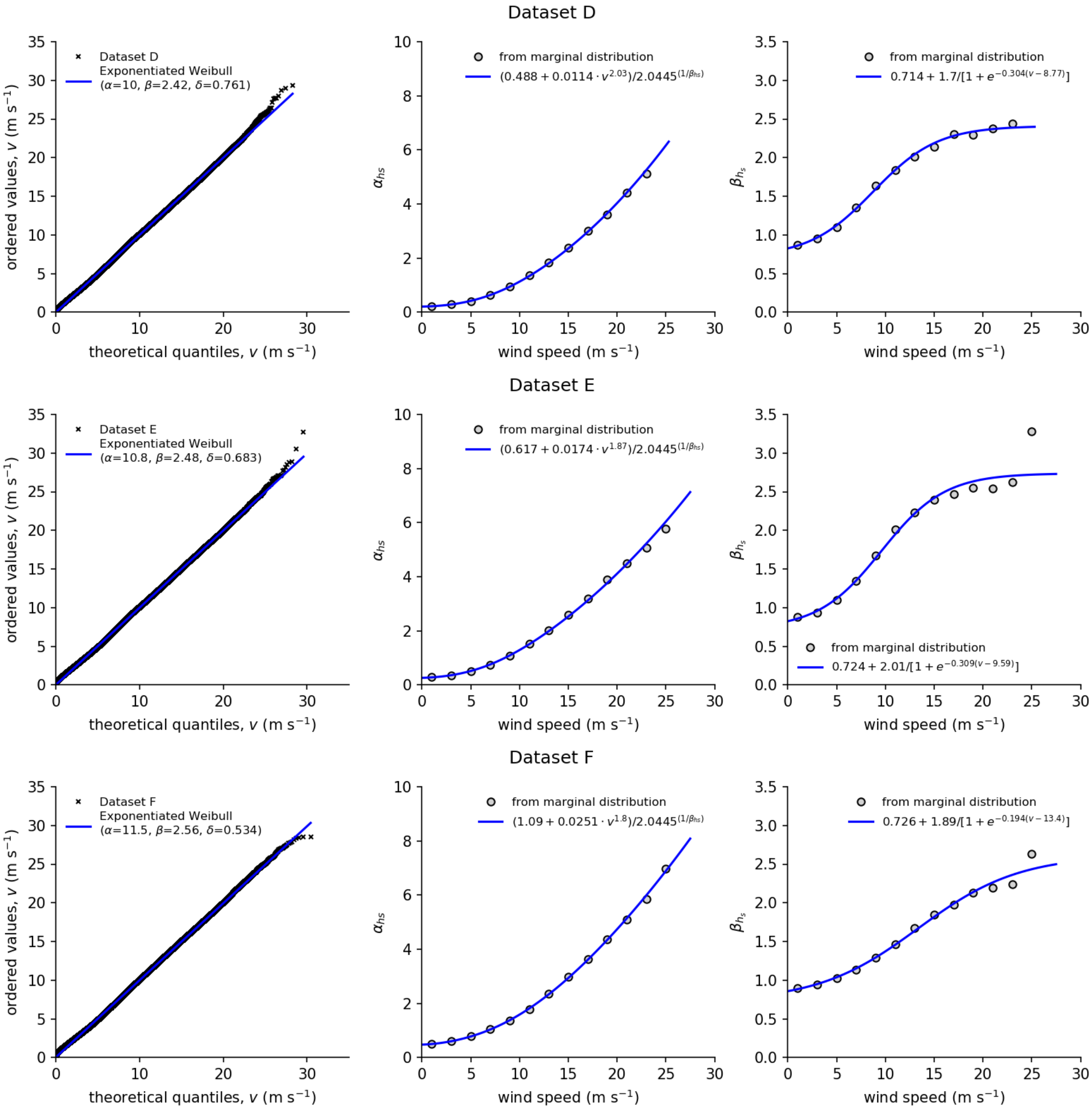

Figure 5. FITS OF THE WIND-WAVE MODEL. THE DEPENDENCE FUNCTION OF THE SCALE PARAMETER, α_{hs}, IS BASED ON THE MEDIAN OF SIGNIFICANT WAVE HEIGHT, $\tilde{h}_s$, WHICH IS MODELLED AS $\tilde{h}_s = c_6 + c_7 v^{c_8}$ WHERE v REPRESENTS WIND SPEED.

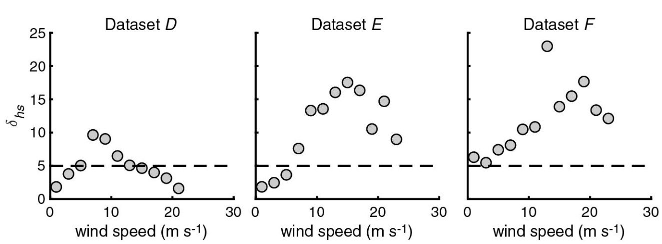

Figure 6. ESTIMATES FOR THE SECOND SHAPE PARAMETER, δ_{hs}, OF THE EXPONENTIATED WEIBULL DISTRIBUTION AT DIFFERENT WIND SPEED INTERVALS. ALTHOUGH THE EXPONENT CHANGES, IN OUR MODEL, WE SET $\delta_{hs} = 5$ TO REDUCE MODEL COMPLEXITY.

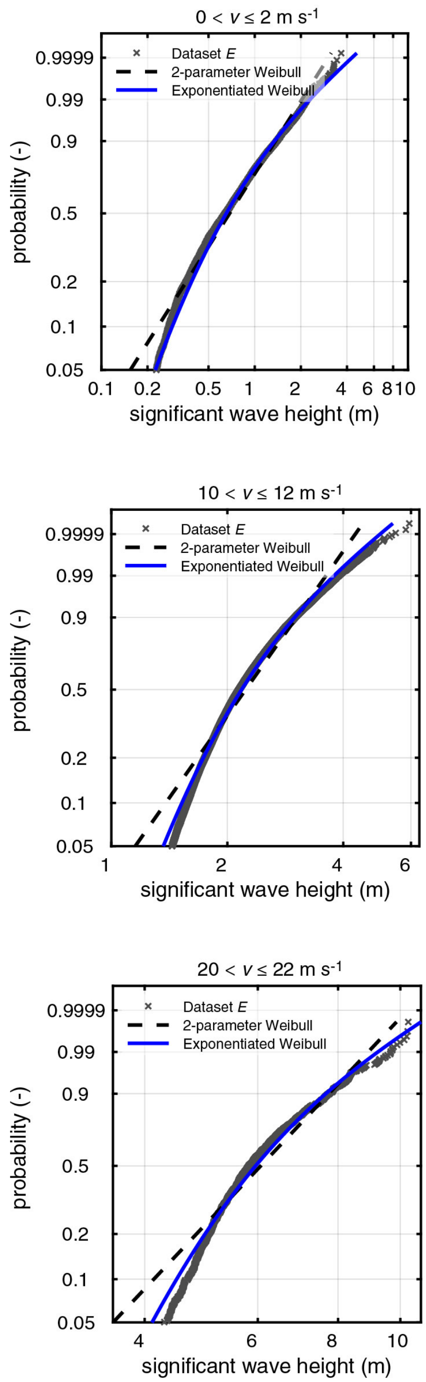

Figure 7. WEIBULL PROBABILITY PLOTS FOR SIGNIFICANT WAVE HEIGHT AT DIFFERENT WIND SPEEDS FOR DATASET E. THE EXPONENTIATED WEIBULL DISTRIBUTION WITH $\delta = 5$ DESCRIBES THE DATA BETTER THAN A 2-PARAMETER WEIBULL DISTRIBUTION.THE OTHER DATASETS SHOWED SIMILAR BEHAVIOR.

ACKNOWLEDGEMENT AND DATA AVAILABILITY

We thank Edward Mackay for useful discussions.

The repository `https://github.com/ahaselsteiner/2020-paper-omae-hierarchical-models` contains the raw data that were used in this study and scripts to reproduce the analysis. Additionally, it contains all contour coordinates in ASCII files. These files were also submitted to the benchmarking exercise on estimating extreme environmental conditions [4].

REFERENCES

[1] International Electrotechnical Commission, 2019. “Wind energy generation systems - Part 3-1: Design requirements for fixed offshore wind turbines”.

[2] DNV GL, 2017. Recommended practice DNVGL-RP-C205: Environmental conditions and environmental loads. Tech. rep.

[3] Ross, E., Astrup, O. C., Bitner-Gregersen, E., Bunn, N., Feld, G., Gouldby, B., Huseby, A., Liu, Y., Randell, D., Vanem, E., and Jonathan, P., 2019. “On environmental contours for marine and coastal design”. *Ocean Engineering*.

[4] Haselsteiner, A. F., Coe, R. G., Manuel, L., Nguyen, P. T. T., Martin, N., and Eckert-Gallup, A., 2019. “A benchmarking exercise on estimating extreme environmental conditions: Methodology & baseline results”. In Proc. 38th International Conference on Ocean, Offshore and Arctic Engineering (OMAE 2019), American Society of Mechanical Engineers (ASME).

[5] Battjes, J. A., 1972. “Long-term wave height distributions at seven stations around the British Isles”. *Deutsche Hydrographische Zeitschrift,* ***25***(4), pp. 179–189.

[6] Nordenström, N., 1973. “A method to predict long-term distributions of waves and wave-induced motions and loads on ships and other floating structures”. PhD thesis, Chalmers University.

[7] Ochi, M. K., 1992. “New approach for estimating the severest sea state”. In 23rd International Conference on Coastal Engineering, American Society of Civil Engineers, pp. 512–525.

[8] Haver, S., 1985. “Wave climate off northern Norway”. *Applied Ocean Research,* ***7***(2), pp. 85–92.

[9] National Data Buoy Center, 2009. Handbook of automated data quality control checks and procedures. Tech. rep.

[10] Groll, N., and Weisse, R., 2017. “A multi-decadal wind-wave hindcast for the North Sea 1949 - 2014: coastDat2”. *Earth System Science Data,* ***9***, pp. 955–968.

[11] Li, L., Gao, Z., and Moan, T., 2015. “Joint environmental data at five European offshore sites for design of combined wind and wave energy devices”. *Journal of Offshore Mechanics and Arctic Engineering,* ***137***, pp. 031901–1 to 031901–16.

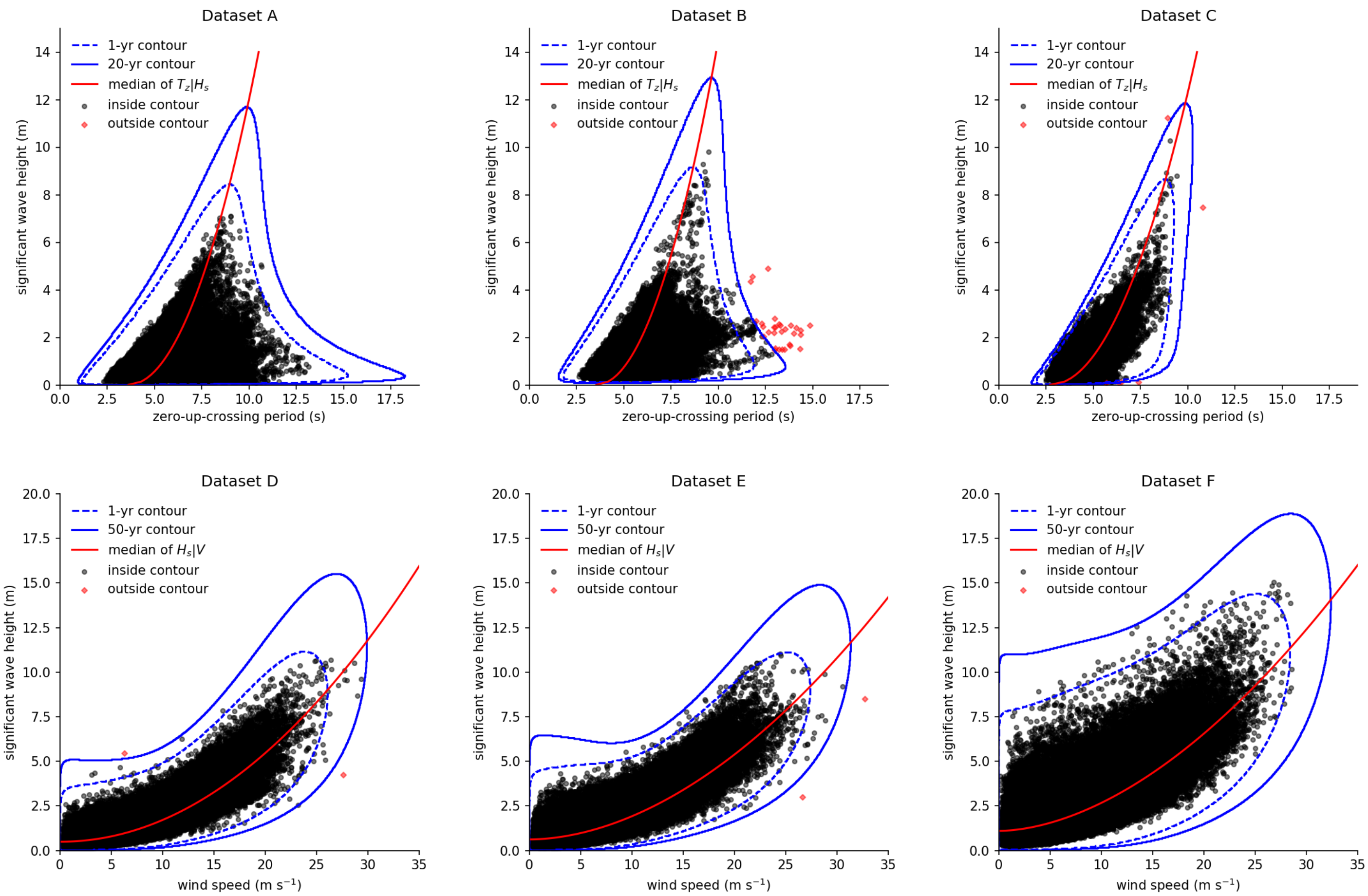

Figure 8. COMPUTED ENVIRONMENTAL CONTOURS.

[12] Horn, J.-T. H., Krokstad, J. R., and Amdahl, J., 2017. "Joint probability distribution of environmental conditions for design of offshore wind turbines". In Proc. 36th International Conference on Ocean, Offshore and Arctic Engineering (OMAE 2017), American Society of Mechanical Engineers (ASME).

[13] Haselsteiner, A. F., and Thoben, K.-D., 2019. "Predicting wave heights for marine design by prioritizing extreme events in a global model". *https://arxiv.org/pdf/1911.12835.pdf*.

[14] Holthuijsen, L. H., 2007. *Waves in oceanic and coastal waters*. Cambridge University Press, Cambridge, United Kingdom.

[15] Haselsteiner, A. F., Lehmkuhl, J., Pape, T., Windmeier, K.-L., and Thoben, K.-D., 2019. "ViroCon: A software to compute multivariate extremes using the environmental contour method". *SoftwareX,* ***9***, pp. 95–101.

[16] Haselsteiner, A. F., Ohlendorf, J.-H., Wosniok, W., and Thoben, K.-D., 2017. "Deriving environmental contours from highest density regions". *Coastal Engineering,* ***123***, pp. 42–51.

[17] Haselsteiner, A. F., Reisenhofer, R., Ohlendorf, J.-H., and Thoben, K.-D., 2019. "Design for extremes: A contour method for defining requirements based on multivariate extremes". In Proc. 22nd International Conference on Engineering Design (ICED19), pp. 1433–1442.

[18] Hyndman, R. J., 1996. "Computing and graphing highest density regions". *The American Statistician,* ***50***(2), pp. 120–126.

[19] Ochi, M., 1998. *Ocean waves: The stochastic approach*. Cambridge University Press, Cambridge, United Kingdom.

[20] Tucker, M. J., and Pitt, E. G., 2001. *Waves in ocean engineering*. Elsevier, Amsterdam, Netherlands.